The eleven-dimensional supermembrane revisited

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ABSTRACT

It is argued that the type IIA 10-dimensional superstring theory is actually a compactified 11-dimensional supermembrane theory in which the fundamental supermembrane is identified with the solitonic membrane of 11-dimensional supergravity. The charged extreme black holes of the 10-dimensional type IIA string theory are interpreted as the Kaluza-Klein modes of 11-dimensional supergravity and the dual sixbranes as the analogue of Kaluza-Klein monopoles. All other p-brane solutions of the type IIA superstring theory are derived from the 11-dimensional membrane and its magnetic dual fivebrane soliton.

The effective field theory of the ten-dimensional type IIA superstring is N=2A supergravity. It has long been appreciated that this field theory is also the effective massless theory for eleven-dimensional supergravity compactified on S^1 ; the ten-dimensional dilaton thereby acquires a natural Kaluza-Klein (KK) interpretation. This leads one to wonder whether the type IIA string theory has an eleven-dimensional interpretation. An obvious candidate is the 11-dimensional supermembrane [1] since the double dimensional reduction of its worldvolume action yields the Green-Schwarz (GS) action of the type IIA superstring [2]. Despite this, the 11-dimensional interpretation of the quantum type IIA superstring is obscure because the dilaton vertex operator is radically different from the graviton vertex operator. In the GS action the dilaton comes from the R-R sector while the graviton comes from the NS-NS sector; there is therefore no obvious KK interpretation of the dilaton in string theory (in the bosonic string the dilaton is usually taken to couple to the worldsheet curvature but this makes the dilaton vertex operator even more dissimilar to the graviton vertex operator). It is possible, however, that this special status of the dilaton is an artefact of perturbation theory. It has recently been realized that some features of the effective field theories of compactified superstring theories, such as invariance under a generalized electromagnetic duality, may also be features of the full non-perturbative string theory even though this is not apparent in perturbation theory [3,4,5]. In this letter I similarly argue that the type IIA 10-dimensional superstring theory actually is a compactified 11-dimensional supermembrane theory.

Before further analysis of this conjecture, some discussion of the status of the 11-dimensional supermembrane is warranted. There is good reason to suppose that the supermembrane spectrum contains massless particles which can be identified as the graviton and other quanta of 11-dimensional supergravity [6]. The principal objection to this conclusion is that there are also reasons [7,8] to believe the spectrum to be continuous, which would preclude a particle interpretation. The physical reason for this is that there is no energy cost to a deformation of the membrane leading to 'spikes' of arbitrary length but zero area, like a fakir's bed

of nails (for the bosonic membrane there is an energy cost at the quantum level due to the Casimir effect, but this Casimir energy cancels for the supermembrane). The possibility of spikes of zero area is of course due to the supposition that the membrane has a core of zero width. A calculation [8] in the context of a first-quantized, regularized, zero-width supermembrane showed that the spectrum is indeed continuous, from zero, and this was widely interpreted as putting an end to the idea of a 'fundamental' supermembrane.

However, evidence was presented in [5] that the fundamental supermembrane should be identified with the solitonic membrane [9] of 11-dimensional supergravity. An additional reason for this identification is that κ -symmetry of the worldvolume action for a supermembrane requires the background fields to satisfy the sourcefree field equations of 11-dimensional supergravity [1]. This is paradoxical if the supermembrane is regarded as the source of the background fields, but the paradox would be resolved if the fundamental supermembrane were to be identified with a membrane solution of the source-free field equations, and the one of [9] is the only candidate. As originally presented this was seen as the exterior solution to a singular surface, which was interpreted as a membrane source, but the singularity can be interpreted equally well as a mere coordinate singularity at an event horizon, through which the source-free exterior solution can be analytically continued [10]. If one accepts the identification of the fundamental and solitonic supermembranes in the fully non-perturbative quantum theory, then it follows that the supermembrane acquires a core of finite size due to its gravitational field in the same way that a 'point' particle actually has a size of the order of its Schwarzschild radius once gravitational effects are included. In this case a 'spike' of a given length has a minimum area and therefore a minimum energy cost. Under these circumstances one would not expect a continuous spectrum. A possible objection to this argument is that it could also be applied to string theory where, however, it is not needed because the spectrum is already discrete in perturbation theory. This may simply be a reflection of the fact that perturbation theory makes sense for strings because of the renormalizability of two-dimensional sigma-models whereas

it does not make sense for membranes because of the non-renormalizability of three-dimensional sigma models. In any case, I shall assume in the following that the fully non-perturbative supermembrane spectrum is discrete for reasons along the above lines. It is perhaps worth mentioning here that a similar argument would be needed to make sense of a 10-dimensional 'fundamental' fivebrane, so that evidence in favour of string-fivebrane duality can be construed as evidence that non-perturbative effects cause the spectrum of a 'fundamental' fivebrane to be discrete, and if this is case for fivebranes then why not for p-branes in general?

The determination of the spectrum of the 11-dimensional supermembrane, given that it is discrete, is impossible in practice, as it is for superstrings when account is taken of interactions and all non-perturbative effects. However, certain features of the spectrum can be reliably ascertained. Among these is the massless spectrum, for which the effective field theory is just 11-dimensional supergravity. This theory reduces to 10-dimensional N=2A supergravity upon compactification on S^1 , but the spectrum in 10-dimensions will then also include the charged massive KK states. These states must also be present in the spectrum of the type IIA superstring if the latter is to be interpreted as a compactified supermembrane, as conjectured here. These states do not appear in perturbation theory but there are extreme black hole solutions of 10-dimensional N=2A supergravity that are charged with respect to the KK U(1) gauge field [11]. Because these solutions preserve half of the supersymmetry there are good reasons (see e.g. [5] and references therein) to believe that their semi-classical quantization of will be exact. I suggest that these states be identified as KK states. I shall now address possible objections to this identification.

First, the mass of a KK state is an integer multiple of a basic unit (determined by the S^1 radius) whereas the mass of an extreme black hole is apparently arbitrary. However, there are also 6-brane solutions of N=2A supergravity [11] that are the magnetic duals of the extreme black holes. It will be shown below that these 6-branes are completely non-singular when interpreted as solutions of the compactified 11-dimensional supergravity. It follows, if the 11-dimensional interpretation

is taken seriously, that the 6-brane solitons must be included as solutions of the ten-dimensional theory and then, by the generalization of the Dirac quantization condition to p-branes and their duals [12], we conclude that in the quantum theory the electric charge of the extreme black holes is quantized. Since their mass is proportional to the modulus of their charge, with a universal constant of proportionality, their mass is also quantized. The unit of mass remains arbitrary, as was the S^1 radius.

Second, it may be objected that whereas the type IIA theory has only one set of charged states coupling to the U(1) gauge field, the compactified supermembrane theory has two: the extreme black hole solutions of the effective 10-dimensional field theory after compactification on S^1 and the KK modes. The two sets of states have identical quantum numbers since the allowed charges must be the same in both cases. It has recently been argued in the context of compactifications of the heterotic [13] and the type II [5] superstrings that KK states should be identified with electrically charged extreme black holes (see also [14]). The reasons advanced for this identification do not obviously apply in the present context but once the principle is granted that this identification is possible it seems reasonable to invoke it more generally. Thus, I conjecture that the resolution of this second objection is that the KK and extreme black hole states of the S^1 compactified 11-dimensional supergravity are not independent in the context of the underlying supermembrane theory. This conjecture is similar to those made recently for the heterotic and type II superstrings but there is a crucial difference; in the string theory case the KK states also appear in the perturbative string spectrum since they result from compactification from the critical dimension, whereas the KK states discussed here do not appear in the perturbative string spectrum because they result from compactification to the critical dimension.

Little more can be said about the spectrum of *particle* states in ten dimensions since only those solutions of the effective field theory that do not break all supersymmetries can yield reliable information about the exact spectrum upon semi-classical quantization, and the only such particle-like solutions are the extreme

electric black holes. However, there are also p-brane solitons of N=2A supergravity which preserve half the supersymmetry and are therefore expected to be exact solutions of type IIA string theory. These should also have an 11-dimensional interpretation. The 6-brane soliton has already been mentioned; we now turn to its 11-dimensional interpretation. Consider the 11-metric

$$ds_{11}^2 = -dt^2 + dy \cdot dy + V(\mathbf{x})d\mathbf{x} \cdot d\mathbf{x} + V^{-1}(\mathbf{x})\left(dx^{11} - \mathbf{A}(\mathbf{x}) \cdot d\mathbf{x}\right)^2, \tag{1}$$

where $dy \cdot dy$ is the Euclidean metric on \mathbb{R}^6 (an infinite planar 6-brane) and $d\mathbf{x} \cdot d\mathbf{x}$ is the Euclidean metric on \mathbb{R}^3 (the uncompactified transverse space). This metric solves the 11-dimensional vacuum Einstein equations, and hence the field equations of 11-dimensional supergravity when all other fields are set to zero, if $\nabla \times \mathbf{A} = \nabla V$, which implies that $\nabla^2 V = 0$. One solution is

$$V = 1 + \frac{\mu}{\rho} \tag{2}$$

where $\rho = \sqrt{\mathbf{x} \cdot \mathbf{x}}$ and μ is a constant. The two-form F = dA is then given by

$$F = \mu \varepsilon_2 , \qquad (3)$$

where ε_2 is the volume form on the unit 2-sphere. The singularity at $\rho = 0$ is merely a coordinate singularity if x^{11} is identified modulo $4\pi\mu$. Thus (1) is a non-singular solution of compactified 11-dimensional supergravity representing a magnetic KK 6-brane. It is an exact analogue in 11 dimensions of the KK monopole in 5 dimensions [16]. Considered as a solution of the effective field theory of tendimensional string theory, the 10-metric, in 'string conformal gauge', is

$$ds_{10}^2 = \left(1 + \frac{\mu}{\rho}\right)^{-\frac{1}{2}} \left[-dt^2 + dy \cdot dy + \left(1 + \frac{\mu}{\rho}\right) d\mathbf{x} \cdot d\mathbf{x} \right]$$
(4)

while the 10-dimensional dilaton field ϕ is given by

$$e^{-2\phi} = \left(1 + \frac{\mu}{\rho}\right)^{\frac{3}{2}}. (5)$$

In terms of the new radial coordinate $r = \rho + \mu$, we have

$$ds_{10}^{2} = \left(1 - \frac{\mu}{r}\right)^{\frac{1}{2}} \left[-dt^{2} + dy \cdot dy \right] + \left(1 - \frac{\mu}{r}\right)^{-\frac{1}{2}} dr^{2} + r^{2} \left(1 - \frac{\mu}{r}\right)^{\frac{3}{2}} d\Omega_{2}^{2}$$

$$e^{-2\phi} = \left(1 - \frac{\mu}{r}\right)^{-\frac{3}{2}},$$
(6)

where $d\Omega_2^2$ is the metric on the unit 2-sphere. This is just the 6-brane solution [11] of 10-dimensional N=2A supergravity.

The remaining p-brane soliton solutions of N=2A supergravity are the string [17], membrane, fourbrane and fivebrane [11]. The string and fourbrane solitons have previously been shown [2,10] to be double-dimensional reductions of, respectively, the 11-dimensional membrane and the 11-dimensional fivebrane [18]. The 10-dimensional membrane and fivebrane differ from their 11-dimensional counterparts simply by the boundary conditions imposed on the solution of the Poisson equation with a point source that always arises in the context of the extreme pbrane solitons, and the ten-dimensional soliton can be viewed as a periodic array of 11-dimensional solitons. Thus, all p-brane solitons of 10-dimensional N=2A supergravity have an 11-dimensional origin. Moreover, since the 11-dimensional fivebrane has a completely non-singular analytic extension through its horizon [15], the 10-dimensional magnetic 4,5 and 6-brane solitons are all completely non-singular when interpreted as solutions of compactified 11-dimensional supergravity. This can be taken as further evidence in favour of an 11-dimensional origin of these solutions of the apparently 10-dimensional type IIA superstring theory. It is perhaps worth remarking that, not surprisingly, there is no similar interpretation of the p-brane solitons of type IIB superstring theory.

It may be objected here that while all of the p-brane solitons of the type IIA superstring may be solutions of an S^1 -compactified supermembrane theory, the two theories differ in that one has an additional fundamental string while the other has an additional fundamental membrane. But this difference disappears once one identifies the fundamental string or membrane with the solitonic ones;

both theories then have exactly the same spectrum of extended objects. In fact, it becomes a matter of convention whether one calls the theory a string theory, a membrane theory, or a p-brane theory for any of the other values of p for which there is a soliton solution; all are equal partners in a p-brane democracy.

In 11-dimensions the fivebrane soliton is the magnetic dual of the 11-dimensional membrane soliton, which has been identified with a fundamental supermembrane. As suggested in [5], one can envisage a dual 11-dimensional fivebrane theory in which the soliton fivebrane is identified with a fundamental 11-dimensional fivebrane. Since the physical (i.e. gauge-fixed) worldvolume action of the latter must be [19] a *chiral* six-dimensional supersymmetric field theory based on the self-dual antisymmetric tensor supermultiplet, it seems possible not only that there is a consistent quantum theory of 11-dimensional supergravity based on the supermembrane, but also that its dual formulation might lead to a solution to the chirality problem that bedevils any attempt to obtain a realistic model of particle physics starting from 11-dimensions.

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